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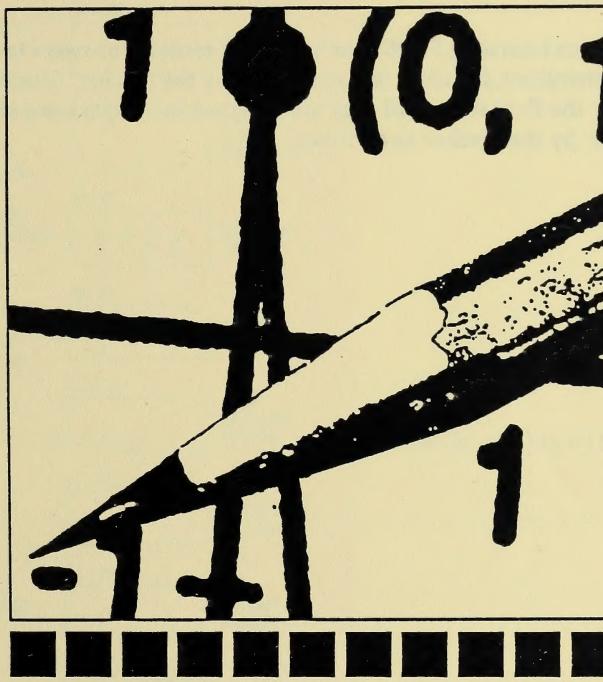
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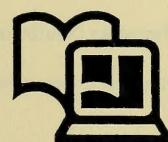
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MATHEMATICS 3

LEARNING FACILITATOR'S MANUAL



UNIT 6: ALGEBRAIC VECTORS AND THEIR APPLICATION



Distance
Learning

Alberta
EDUCATION

Note

This Mathematics Learning Facilitator's Manual contains answers to teacher-assessed assignments and the final test; therefore, it should be kept secure by the teacher. Students should not have access to these assignments or the final tests until they are assigned in a supervised situation. The answers should be stored securely by the teacher at all times.

Mathematics 31
Learning Facilitator's Manual
Unit 6
Algebraic Vectors and Their Application
Alberta Distance Learning Centre
ISBN No. 0-7741-0169-5

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3).

from $M(3, -2, 1)$ to $N(5, 0, 6)$.

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This Mathematics Learning Facilitator's Manual is intended for use in the classroom. It is not to be used for the final test; therefore, it should be kept separate from the student's workbooks. It is to be used for assignments or the final tests until the teacher has stored securely by the teacher at all times.

Assess

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Topic 1: Length of a Vector

② 1. Find the distance between the points $A(3, -2)$ and $B(-1, -3)$.

$$\begin{aligned} d &= \sqrt{(3+1)^2 + (-2+3)^2} \\ &= \sqrt{16+1} \\ &= \sqrt{17} \end{aligned}$$

③ 2. Find the length of the vector represented by the line segment from $M(3, -2, 1)$ to $N(5, 0, 6)$.

$$\begin{aligned} d &= \sqrt{(5-3)^2 + (0+2)^2 + (6-1)^2} \\ &= \sqrt{4+4+25} \\ &= \sqrt{33} \end{aligned}$$

(5)

3. Determine if the three points $P(0, 1, 0)$, $Q(2, 3, 1)$, and $R(4, 5, 2)$ are collinear.

$$\begin{aligned}PQ &= \sqrt{(2-0)^2 + (3-1)^2 + (1-0)^2} \\&= \sqrt{9} \\&= 3\end{aligned}$$

$$\begin{aligned}QR &= \sqrt{(4-2)^2 + (5-3)^2 + (2-1)^2} \\&= \sqrt{9} \\&= 3\end{aligned}$$

$$\begin{aligned}PR &= \sqrt{(4-0)^2 + (5-1)^2 + (2-0)^2} \\&= \sqrt{36} \\&= 6\end{aligned}$$

$$PR = PQ + QR$$

Therefore, P , Q , and R are collinear.

(5)

4. Determine if the three points $A(3, 5)$, $B(0, 4)$, and $C(1, 1)$ form a right triangle.

$$\begin{aligned}AB &= \sqrt{(3-0)^2 + (5-4)^2} \\&= \sqrt{10}\end{aligned}$$

$$\begin{aligned}BC &= \sqrt{(1-0)^2 + (1-4)^2} \\&= \sqrt{10}\end{aligned}$$

$$\begin{aligned}AC &= \sqrt{(3-1)^2 + (5-1)^2} \\&= \sqrt{20}\end{aligned}$$

Since $AC^2 = AB^2 + BC^2$, $\triangle ABC$ is a right triangle.

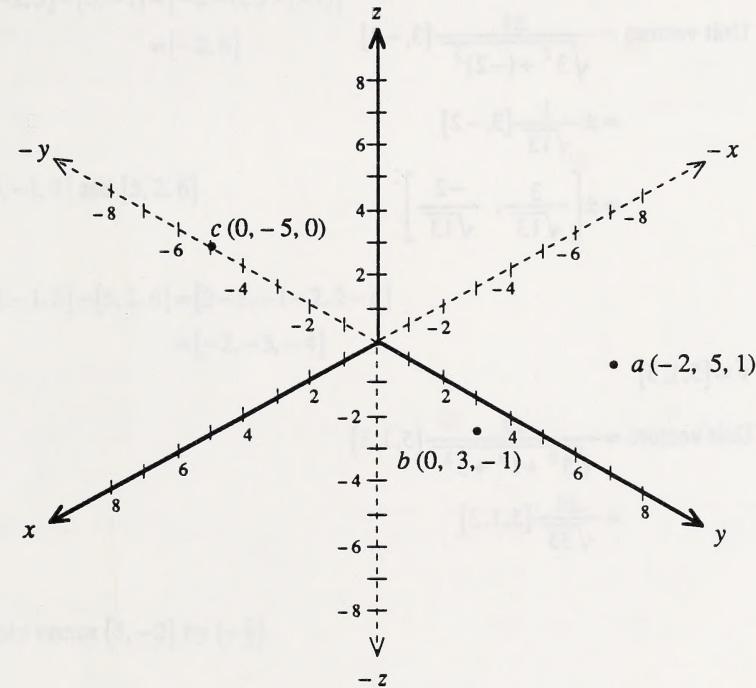
(3)

5. Plot the following points.

a. $(-2, 5, 1)$

b. $(0, 3, -1)$

c. $(0, -5, 0)$



(2)

6. a. The characteristic of a point on the xz -plane is $y = 0$.b. The characteristic of a point on the z -axis is $x = y = 0$.**Topic 1** marks

Topic 2: Operations Defined on Algebraic Vectors

1. Find two unit vectors collinear with each of the following vectors.

(4)

a. $\vec{v} = [3, -2]$

$$\begin{aligned}\text{Unit vectors} &= \frac{\pm 1}{\sqrt{3^2 + (-2)^2}} [3, -2] \\ &= \pm \frac{1}{\sqrt{13}} [3, -2] \\ &= \pm \left[\frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right]\end{aligned}$$

b. $\vec{v} = [5, 1, 3]$

$$\begin{aligned}\text{Unit vectors} &= \frac{\pm 1}{\sqrt{5^2 + 1^2 + 3^2}} [5, 1, 3] \\ &= \frac{\pm 1}{\sqrt{35}} [5, 1, 3]\end{aligned}$$

(4)

2. Find the sums of the following vectors.

a. $[-2, 5]$ and $[3, -7]$

$$\begin{aligned}[-2, 5] + [3, -7] &= [-2 + 3, 5 - 7] \\ &= [1, -2]\end{aligned}$$

b. $[0, 5, 1]$ and $[3, 2, -8]$

$$\begin{aligned}[0, 5, 1] + [3, 2, -8] &= [0 + 3, 5 + 2, 1 - 8] \\ &= [3, 7, -7]\end{aligned}$$

(4)

3. Subtract the second vector from the first vector.

a. $[-2, 5]$ and $[0, -1]$

$$\begin{aligned} [-2, 5] - [0, -1] &= [-2 - 0, 5 - (-1)] \\ &= [-2, 6] \end{aligned}$$

b. $[3, -1, 2]$ and $[5, 2, 6]$

$$\begin{aligned} [3, -1, 2] - [5, 2, 6] &= [3 - 5, -1 - 2, 2 - 6] \\ &= [-2, -3, -4] \end{aligned}$$

(2)

4. Multiply vector $[5, -2]$ by $(-\frac{2}{3})$.

$$\left(-\frac{2}{3}\right)[5, -2] = \left[-\frac{10}{3}, \frac{4}{3}\right]$$

(5)

5. Find x and y if $5[x, 3] + 2[-3, y] = [4, 1]$.

$$5x - 6 = 4$$

$$5x = 10$$

$$x = 2$$

$$15 + 2y = 1$$

$$2y = -14$$

$$y = -7$$

(5)

6. Prove that $\{[0, 5] + [-3, 1]\} + [-5, -2] = [0, 5] + \{[-3, 1] + [-5, -2]\}$.

LS	RS
$[-3, 6] + [-5, -2]$	$[0, 5] + [-8, -1]$
$[-8, 4]$	$[-8, 4]$
LS	= RS

(6)

7. Determine x , y , and z if $2[3, y, 0] - 5[x, -2, 2] + [1, 3, -4] = [y, z, x]$.

$$\begin{aligned} 6 - 5x + 1 &= y \\ 5x + y - 7 &= 0 \end{aligned} \quad \textcircled{1}$$

$$\begin{aligned} 2y + 10 + 3 &= z \\ 2y - z + 13 &= 0 \end{aligned} \quad \textcircled{2}$$

$$\begin{aligned} 0 - 10 + (-4) &= x \\ x &= -14 \end{aligned} \quad \textcircled{3}$$

Substitute $\textcircled{3}$ in $\textcircled{1}$.

$$\begin{aligned} 5(-14) + y - 7 &= 0 \\ y &= 77 \end{aligned}$$

Substitute $y = 77$ in $\textcircled{2}$.

$$\begin{aligned} 2(77) - z + 13 &= 0 \\ z &= 154 + 13 \\ &= 167 \end{aligned}$$

Topic 2

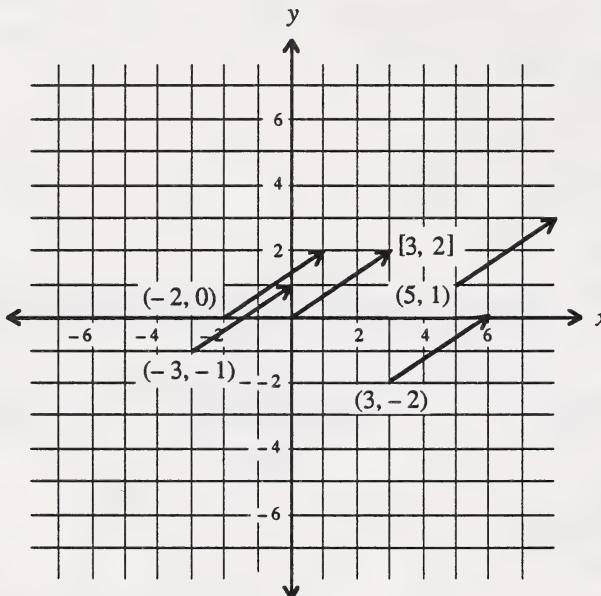
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Topic 3: Two- and Three-Dimensional Vectors in Algebraic Form

(4)

1. If the position (basic) vector is $[3, 2]$, draw an equivalence class of vectors with the following initial points.

- $(5, 1)$
- $(-2, 0)$
- $(-3, -1)$
- $(3, -2)$



(2)

2. Find the position (basic) vector which corresponds to the geometric vector \overrightarrow{AB} determined by $A(3, 7)$ and $B(5, -2)$.

The coordinates of this position (basic) vector are $[5 - 3, -2 - 7] = [2, -9]$.

(2)

3. Find the coordinates of the position (basic) vector which corresponds to the geometric vector \overrightarrow{xy} determined by x which is at $(3, 2, -5)$ and y which is at $(-1, 0, -2)$.

The coordinates of this position (basic) vector are $[-1-3, 0-2, -2+5] = [-4, -2, 3]$.

(2)

4. $[3, -1]$ is a position (basic) vector. Find the terminal point of vector \overrightarrow{PQ} which is equivalent to the position (basic) vector. The initial point P is $(4, 5)$.

The terminal point is $(4+3, -1+5) = (7, 4)$.

Topic 3

_____ marks

Topic 4: Collinear and Coplanar Algebraic Vectors

②

1. Find the set of vectors which is parallel to each of the following:

a. $[-5, 1]$

b. $[3, 7, -2]$

$$\{k[-5, 1] \mid k \in R\}$$

$$\{k[3, 7, -2] \mid k \in R\}$$

④

2. Find the set of vectors which is parallel to vector \overrightarrow{PQ} if P and Q are the following points:

a. $P(3, -2)$ $Q(6, 4)$

$$\overrightarrow{PQ} = [6 - 3, 4 + 2] = [3, 6]$$

Thus, the set of vectors is $\{k[3, 6] \mid k \in R\}$.

b. $P(3, 5, -1)$ $Q(0, 2, 4)$

$$\overrightarrow{PQ} = [0 - 3, 2 - 5, 4 + 1] = [-3, -3, 5]$$

Thus, the set of vectors is $\{k[-3, -3, 5] \mid k \in R\}$.

④ 3. $[4, 3, -6]$ and $[2, \frac{3}{2}, -3]$ are two vectors. Are they collinear? Explain.

Yes, they are collinear because $[4, 3, -6] = 2[2, \frac{3}{2}, -3]$. They are scalar multiples of each other.

⑧ 4. $[3, 5, k]$ and $[-6, l, 4]$ are two collinear vectors. Find k and l .

$$\begin{aligned} -6 &= (-2)(3) \\ [-6, l, 4] &= (-2)[3, 5, k] \\ &= [-6, -10, -2k] \end{aligned}$$

$$\therefore l = -10$$

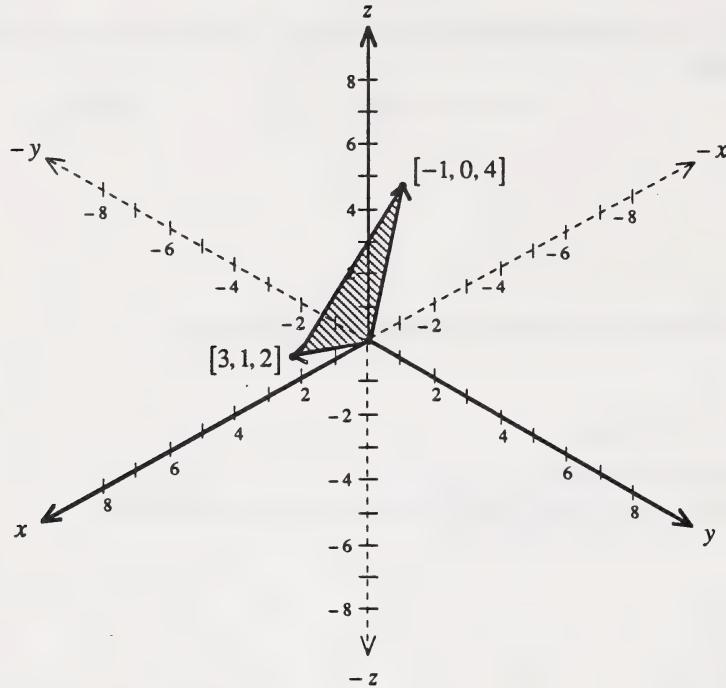
$$\begin{aligned} -2k &= 4 \\ \therefore k &= -2 \end{aligned}$$

② 5. $\{k[-5, 0, -3] \mid k \in R\}$ determines the line in three-space through the origin. Find the coordinates of two points on the line.

Answers will vary.

(4)

6. The two position (basic) vectors $\vec{u} = [3, 1, 2]$ and $\vec{v} = [-1, 0, 4]$ determine a plane through the origin. Sketch the plane. (Shade the portion of the plane between the vectors.)



(3)

7. Are the vectors $[3, -5]$, $[9, -15]$, and $[-2, 7]$ collinear or coplanar?

$$[9, -15] = 3[3, -5]$$

These two vectors are collinear, but $[-2, 7]$ is not collinear with either of these.

Noncollinear vectors in V_2 are coplanar.

(7)

8. Are the vectors $[-1, -2, -3]$, $[0, 2, 1]$, and $[-2, 4, -2]$ collinear, coplanar, or noncoplanar?

No two vectors are collinear.

$$\begin{aligned} \text{Let } [-2, 4, -2] &= k[-1, -2, -3] + l[0, 2, 1] \\ &= [-k, -2k, -3k] + [0, 2l, l] \end{aligned}$$

$$-k = -2$$

$$k = 2$$

$$-2k + 2l = 4$$

$$-4 + 2l = 4$$

$$2l = 8$$

$$\therefore l = 4$$

Substitute $k = 2$ and $l = 4$ in $-3k + l = -2$ as a check.

$$\begin{aligned} \text{LS} &= -3(2) + 4 \\ &= -2 \\ \therefore \text{LS} &= \text{RS} \end{aligned}$$

Thus, $[-2, 4, -2]$ is a linear combination of the other two. They are coplanar.

(6)

9. Are the vectors $[3, 5, 7]$, $[0, -2, 1]$, and $[-1, 1, 3]$ collinear, coplanar, or noncoplanar?

By inspecting the vectors, no two vectors are collinear.

$$\begin{aligned} \text{Let } [3, 5, 7] &= k[0, -2, 1] + l[-1, 1, 3]. \\ &= [0, -2k, k] + [-l, l, 3l] \end{aligned}$$

$$\begin{aligned} 3 &= 0 - l & (1) \\ 5 &= -2k + l & (2) \\ 7 &= k + 3l & (3) \end{aligned}$$

From (1), $l = -3$.
Substitute this in (2).

$$\begin{aligned} 5 &= -2k - 3 \\ -2k &= 5 + 3 \\ k &= -4 \end{aligned}$$

Substitute $l = -3$ and $k = -4$ in (3).

$$\begin{aligned} -4 + 3(-3) &= -4 - 9 \\ &= -13 \\ 7 &\neq -13 \end{aligned}$$

The vectors are noncoplanar.

Topic 4

_____ marks

N.L.C.-B.N.C.



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